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Vibration Response to Measure Properties of a System. We often measure the natural frequency and damping coefficient for a mode of vibration in a structure or component, by measuring the forced vibration response of the system. It is how this is done. We find some way to apply the system to the system (base excitation might work; or you can apply a force using some kind of actuator, or you could deliberately mount an unbalanced rotor on the system). Then, we mount accelerometers on our system, and use them to measure the displacement of the structure, at the point where it is being excited, as a function of frequency. We then plot a graph, which usually looks something like the picture on the right. We read off the maximum response  $A$ , and draw a horizontal line at amplitude  $A$ . Finally, we measure the frequencies  $f_1$  and  $f_2$ , and as shown in the picture. We define the bandwidth of the response as  $\Delta f = f_2 - f_1$ . Like the logarithmic decrement, the bandwidth of the forced harmonic response is a measure of the damping in a system. It turns out that we can estimate the natural frequency of the system and its damping coefficient using the following formulae. The formulae are accurate for small  $\Delta f$  - say,  $\Delta f < 0.1 f_n$ . To understand the origin of these formulae, recall that the amplitude of vibration due to external forcing is given by  $A = \frac{F_0}{m \sqrt{(\omega_n^2 - \omega^2)^2 + 2\zeta\omega_n\omega}}$ . We can find the frequency at which the amplitude is a maximum by differentiating with respect to  $\omega$ , setting the derivative equal to zero and solving the resulting equation for frequency. It turns out that the maximum amplitude occurs at a frequency  $\omega = \omega_n \sqrt{1 - 2\zeta^2}$ . For small  $\zeta$ , we see that  $\omega \approx \omega_n$ . Next, to get an expression relating the bandwidth  $\Delta \omega$  to  $\zeta$ , we first calculate the frequencies  $\omega_1$  and  $\omega_2$ . Note that the maximum amplitude of vibration can be calculated by setting  $\frac{dA}{d\omega} = 0$ , which gives  $\omega = \omega_n \sqrt{1 - 2\zeta^2}$ . Now, at the two frequencies of interest, we know  $A$ , so that  $\omega_1$  and  $\omega_2$  must be solutions of the equation  $\frac{F_0}{m \sqrt{(\omega_n^2 - \omega^2)^2 + 2\zeta\omega_n\omega}} = A$ . Rearrange this equation to see that  $(\omega_n^2 - \omega^2)^2 + 2\zeta\omega_n\omega = \frac{F_0}{mA}$ . This is a quadratic equation for  $\omega^2$  and has solutions  $\omega^2 = \omega_n^2 \pm \omega_n \sqrt{\omega_n^2 - 2\zeta^2 \omega_n^2}$ . Expand both expressions in a Taylor series about  $\omega_n$  to see that  $\omega_1 \approx \omega_n - \zeta\omega_n$  and  $\omega_2 \approx \omega_n + \zeta\omega_n$ , so finally, we confirm that  $\Delta \omega = 2\zeta\omega_n$ . 5.4.7 Example Problems in Forced Vibrations Example 1: A structure is idealized as a damped springmass system with stiffness 10 kN/m; mass 2Mg; and dashpot coefficient 2 kNs/m. It is subjected to a harmonic force of amplitude 500N at frequency 0.5Hz. Calculate the steady state amplitude of vibration. Start by calculating the properties of the system: Now, the list of solutions to forced vibration problems gives For the present problem: Substituting numbers into the expression for the vibration amplitude shows that Example 2: A car and its suspension system are idealized as a damped springmass system, with natural frequency 0.5Hz and damping coefficient 0.2. Suppose the car drives at speed V over a road with sinusoidal roughness. Assume the roughness wavelength is 10m, and its amplitude is 20cm. At what speed does the maximum amplitude of vibration occur, and what is the corresponding vibration amplitude? Let  $s$  denote the distance traveled by the car, and let  $L$  denote the wavelength of the roughness and  $H$  the roughness amplitude. Then, the height of the wheel above the mean road height may be expressed as  $y = H \sin(\frac{s}{L})$ . Noting that  $\dot{y} = v$ , we have that  $\ddot{y} = \frac{v^2}{L} \sin(\frac{s}{L})$ , i.e., the wheel oscillates vertically with harmonic motion, at frequency  $\frac{v}{L}$ . Now, the suspension has been idealized as a springmass system subjected to base excitation. The steady state vibration is  $y = A \sin(\frac{v}{L}t)$ . For light damping, the maximum amplitude of vibration occurs at around the natural frequency. Therefore, the critical speed follows from  $\frac{v}{L} = \omega_n$ . Note that  $K=1$  for base excitation, so that the amplitude of vibration at  $\omega_n$  is approximately  $A = \frac{H}{2\zeta}$ . Note that at this speed, the suspension system is making the vibration worse. The amplitude of the car's vibration is greater than the roughness of the road. Suspensions work best if they are excited at frequencies well above their resonant frequencies. Example 3: The suspension system discussed in the preceding problem has the following specifications. For the roadway described in the preceding section, the amplitude of vibration may not exceed 35cm at any speed. At 55 miles per hour, the amplitude of vibration must be less than 10cm. The car weighs 3000lb. Select values for the spring stiffness and the dashpot coefficient. We must first determine values for  $k$  and  $c$  that will satisfy the design specifications. To this end: (i) The specification requires that  $A = 0.35$  m at resonance. Examine the graph of  $A$  shown with the solutions to the equations of motion. Recall that  $K=1$  for a base excited springmass system. Observe that, with  $\omega_n = 1.75$ , the amplification factor never exceeds 1.75. (ii) Now, the frequency of excitation at 55mph is  $\omega = 2.47$ . We must choose system parameters so that, at this excitation frequency,  $A = 0.1$  m. Examine the graph showing the response of a base excited springmass system again. We observe that, for  $\omega = 2.47$ ,  $A = 0.1$  m. Therefore, we pick  $k = 1.75^2 \times 10^4$  N/m. Finally, we can compute properties of the system. We have that  $\zeta = \frac{c}{2m\omega_n}$ . Similarly, An oscillation is simply the periodic back-and-forth motion between two positions or states. We have seen many real-life scenarios of such motion in daily life, such as the side-to-side swing of a pendulum or the up-and-down motion of a spring with a weight show oscillation. Due to the absence of 'eternal motion' in physical experiments, we encounter various types of oscillations, including free, forced, and damped oscillations. The derived mathematical expressions of such motion become very useful in tailoring the efficiency of a mechanical system. Periodic motion repeats itself in a regular cycle, like a sine wave—a wave with eternal motion. Oscillation depends upon the period, frequency and amplitude of oscillation. An oscillating movement occurs around an equilibrium point. Here, we will learn more about free, forced, and damped oscillations and solve some related questions. [Click Here for Sample Questions] Oscillation is the periodic fluctuation of an object about its mean value or between two fixed states. The point at which the body starts moving is called the mean or equilibrium position. It is a time-dependent quantity that is calculated in terms of a condition of equilibrium. The motion produces a continuous, repeated, alternating waveform without any input. The unit of oscillation is Hertz. We can also induce electrical oscillations in a circuit through a device called an oscillator. The oscillograph and oscilloscope are two more instruments based on oscillation principles. Oscillators might be mechanical or electrical, but they all function in the same way. It occurs in everything from tides and the pendulum of a clock to our decision-making process. Damped, free and forced oscillation depends upon three quantities, which are as follows: Physical Quantities Definition S.I Unit Time Period The time taken by the oscillating body to complete one complete oscillation is called the period of oscillation. Second Frequency The number of oscillations completed by oscillating body per second is called the frequency of oscillation. Hertz Amplitude The maximum displacement of the oscillating body with respect to the mean position is called the amplitude of oscillation. Metre Oscillation Damped, Free and Forced Oscillation [Click Here for Sample Questions] Oscillation can be classified into three categories: Damped, free and forced oscillation. Damped Oscillation Damped oscillation is a type of oscillation in which the amplitude decreases with respect to time. The reduction in amplitude is due to external variables like friction or air resistance, which cause damping, resulting in energy loss from the system. An object is called damped when there is a difference between the applied restoring force and the restraining force acting on the object. The fading oscillations of a pendulum are an example. Underdamped and overdamped oscillation are two different types of damped oscillation. The situation when the amplitude of oscillation never becomes zero even after significant deceleration is called underdamped oscillation. The situation when the amplitude of oscillation reaches zero due to a significant reduction in amplitude is called overdamped oscillation. Free Oscillation A free oscillation occurs when a body vibrates at its own frequency. Without any external force to set the oscillation, this oscillation has a constant amplitude and time. Free oscillation does not experience any form of damping. It is a type of oscillation that exhibits natural frequency, which corresponds to the constant amplitude, energy, and time period. The vibrations in a tuning fork are an example. Forced Oscillation A forced oscillation occurs when any object oscillates as a result of an external periodic force. Since other external energy is supplied to it, the amplitude of the oscillation is damped but remains constant. It is also known as a driven or forced harmonic oscillator. Example of Forced Oscillation For instance, when you're playing with a toy that requires you to sustain an object using an elastic band dangling from your finger. If you keep your finger motionless, the object will bounce up and down with a modest bit of dampening at first. The thing will follow your finger as you move it up and down. As you raise the frequency with which you move your fingers, the item responds by oscillating with increasing amplitude. Calculation of Damped, Free and Forced Oscillation [Click Here for Sample Questions] When a body is in the oscillatory motion then three terms are used in the calculation namely time period, frequency and amplitude. Each succeeding string vibration takes the same amount of time as the preceding one. So, we get 'period' (T), the amount of time it takes to complete one oscillation. Consider l l be the length of the string, then its time period is given by  $T = 2\pi\sqrt{\frac{l}{g}}$  Where, T : time period of oscillation L : length of string g : acceleration due to gravity The relation between time period and frequency is given by frequency ( $\eta$ ) = 1/time period (T)  $\eta = 1/T$  Examples of Oscillations [Click Here for Sample Questions] Oscillatory motion is prominent in- Pendulum Clock. Tuning Fork. Swing. Flapping of Wings. A freely hanging Bob. String Musical Instruments. Spring Toy. Alternating Current. Simple Harmonic Motion [Click Here for Sample Questions] Simple harmonic motion is defined as a repeated back-and-forth movement about a central position with the maximum displacement on one side equal to the maximum displacement on the other. The time interval between each full vibration is the same. It is characterized by a variable acceleration that is proportionate to the displacement from the equilibrium point. It is a special case of oscillatory motion. Linear Simple Harmonic Motion and Angular Simple Harmonic Motion are two types of Simple Harmonic Motion. Furthermore, the time period between complete vibrations is constant and independent of the greatest displacement size. Mathematically, it can be represented as:  $F = -kx$  where F stands for force x stands for displacement k stands for a constant or restoring force constant. The vibrating of a mass coupled to a vertical spring, the other end of which is anchored in a ceiling, is an example of a simple harmonic oscillator. The spring is at its most tensioned at maximum displacement  $x$ , which forces the mass upward. The spring reaches its greatest compression at maximum displacement  $+x$ , which forces the mass downward again. At either side of maximum displacement, the force is largest and directed toward the equilibrium position, the mass's velocity (v) is zero, its acceleration is at a maximum, and the mass changes direction. In the equilibrium position, the velocity is at its highest, and the acceleration (a) is zero. An example of Simple Harmonic Oscillator Things to Remember Oscillation refers to the periodic back-and-forth movement of something between two positions or states. Resonance is the phenomenon of driving a system with a frequency equal to its natural frequency. The forced oscillation occurs when a body oscillates as a result of an external periodic force. A free oscillation occurs when a body vibrates at its frequency. Damped oscillation is a type of oscillation that reduces with time. Ques. Can a motion be periodic and not oscillatory? Explain. (2 marks) Ans: Although every oscillatory motion is periodic, not all periodic motions are oscillatory. Circular motion is a periodic but not oscillatory motion. The difference between oscillations and vibrations is insignificant. Ques. What is resonance, and how can it be detected? (2 marks) Ans: When a system can store and quickly transfer energy between two or more separate storage modes (such as kinetic and potential energy in the case of a basic pendulum), it is said to be in resonance. Some systems have numerous resonance frequencies that are distinct. Ques. Is it possible for a motion to be oscillatory while not being simple harmonic? Give a reasoned explanation. (2 marks) Ans: Yes, when a ball is dropped from a height onto a completely elastic surface, the motion is oscillatory but not simple harmonic since the restoring force  $F = mg = \text{constant}$  rather than  $F \propto -x$ , which is a need for S.H.M. Ques. What are the conditions for simple harmonic motion? (2 marks) Ans: The conditions for simple harmonic motion are as follows: The system must be subjected to an elastic restoring force. Inertia is required in the system. The system's acceleration should be proportional to its displacement and always point to the mean location. Ques. How does the damping affect amplitude in forced oscillation? (3 marks) Ans: An under-damped oscillator's amplitude diminishes exponentially. Damping is the process of reducing the magnitude of oscillations due to energy dissipation. The bigger the amplitude of the induced oscillations towards resonance, the less damping a system has. A system's response to varied driving frequencies is broader the more damping it has. Ques. When can resonance be observed? (2 marks) Ans: When a system can store and transmit energy between two or more separate storage modes (for example, kinetic and potential energy in the case of a basic pendulum), it is said to be in resonance. The resonance frequencies of certain systems are many and different. Ques. Explain demonstration of resonance with tuning forks? (2 marks) Ans: The times of the tuning fork vibrate at their natural frequency, causing sound waves to impinge on the resonance tube's opening. The impinging sound waves from the tuning fork cause the air within the resonance tube to vibrate at the same frequency. Ques. What happens when the oscillation is damped? (2 marks) Ans: Non-conservative forces diminish the energy of damped harmonic oscillators. Without overshooting, critical damping brings the system to equilibrium as quickly as possible. In an underdamped system, the equilibrium position will oscillate. Ques. A spring with a 2500 N/m spring constant is crushed by 0.87m. What is the total amount of potential energy that has been generated? (3 marks) Ans. The potential energy of a spring is given by : PE = 1/2kx<sup>2</sup>. Putting the value of k, spring constant, and x, displacement in the above equation, we get : PE = ½ (2500) (0.87)<sup>2</sup> PE = 1087.5 J Ques. A 0.7kg mass is linked to one of the springs, which oscillates with a 5 s period. What is the frequency of the event? (2 marks) Ans. The link between frequency and period is unaffected by mass. The equation that describes this relationship is: f = 1/T The frequency will be above the reciprocal of the period. f = 1/T => f = 1/5=> f = 0.2 Hz Ques. A spring with a 500 N/m spring constant is crushed by 0.17m. What is the total amount of potential energy that has been generated? (3 marks) Ans. The potential energy of a spring is given by : PE = 1/2kx<sup>2</sup> Putting the value of k, spring constant, and x, displacement in the above equation, we get : PE = ½ (500) (0.17)<sup>2</sup> PE = 42.5 J